第9卷　第5期
2001年10月

光学　精密工程
OPTICS AND PRECISION ENGINEERING

Vol. 9, No. 5
Oct., 2001

Article ID: 1004-924X(2001)05-0458-09

Geometric Theory for the Design of Multielement Optical Systems

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Abstract: To establish a theoretical basis for providing a better design method of multielement optical systems, we have developed a third- order geometric theory of a plane- symmetric multielement optical system that consists of a planar light source, an arbitrary number of ellipsoidal gratings, and an image plane. Analytic formulas of spot diagrams are derived for the system by analytically following a ray- tracing formalism. With these formulas, coma, spherical aberration, and resultant aberration are discussed. To make the theory practical, we determine the aberration coefficients numerically, rather than analytically, with the aid of ray tracing that takes into account the angular distribution of rays originating from a given light source. A merit function is defined so as to represent closely the variance of the spots formed when an infinite number of rays are traced and to take into account the dimensions of the source and the last optical element. The theory is also applicable to mirror- grating or mirror systems.

Key words: optical systems; geometry; ray-tracing;
CLC number: TH703　　**Document code:** A

1 Introduction

The current trend in extreme ultraviolet (EUV) lithography and sophisticated synchrotron radiation instrumentation is the use of an optical system that consists of several advanced optical elements, such as accurately figured aspheric mirrors and varied line spacing gratings, to achieve higher spatial and/or spectral resolution. To design and evaluate such a system, it is important to develop a practical design theory that takes into account all the elements of the system from its source to the final image plane. Many of the current design methods utilize ray tracing because no analytic design theory is available at present even for a simple triple- element optical system, so long as it includes a grating. Ray tracing is certainly a powerful tool for designing and evaluating a system consisting of several advanced optical elements. On the other hand, ray tracing is capable of giving a physical insight only to a limited extent. Moreover, ray tracing does not permit individual analysis of coma and spherical aberration in the image in a similar manner to Seidel aberrations of lens systems.

Received date: 2001- 08- 29; **revised date:** 2001- 10- 08

In view of the current trend in EUV/SXR instrumentation and the present status of the design methods, we have felt the need of an aberration theory for a multielement optical system containing advanced elements and an extended source that emits rays of a certain angular distribution. As a step toward such a theory, we have extended our third-order geometric theory of a double-grating system[1-2] to include a plane-symmetric multigrating system that consists of a planar light source, more than two advanced-type ellipsoidal gratings and an image plane. The optical system to be treated in this paper and its coordinate systems are explained in section 2. The complete ray-tracing formalism for a multigrating system is described in section 3, and the third-order analytic formulas for the spot diagrams are derived in section 4, and the equations of aberration curves are given in section 5. Also presented in section 6 is a merit function necessary for optimization of the system.

2 Multielement optical system

Referring to Fig. 1 we consider a plane-symmetric optical system that consists of a planar light source S, ellipsoidal gratings G_1, G_2, …, G_i, …, G_n, and an image plane Σ. (This system also represents a mirror-grating system or a mirror system that includes any of ellipsoidal, spherical, cylindrical, and plane mirrors and/or gratings as its elements.) In this system the elements are arranged so that the normals to S and G_i at their respective centers O_S and O_i lie in a common plane called meridional plane and O_1 situates on the normal to S at O_S. The distances O_SO_1, O_1O_2, O_2O_3, …, $O_{i-1}O_i$, …, and $O_{n-1}O_n$ are denoted by r_1, r_2, r_3, …, r_i, …, and r_n, respectively. The incident principal ray O_SO_1 is diffracted by G_1 toward O_2, and the diffracted principal ray O_1O_2 of wavelength λ in m_1th order is further diffracted by G_2. This diffracted principal ray of λ in m_2th order meets G_3 at O_3. The principal ray of λ goes through O_is successively, and finally the principal ray diffracted by G_n at O_n meets Σ at a point O, which also lies in the meridional plane. The image plane Σ is perpendicular to the diffracted principal ray O_nO, and the distance O_nO is denoted by r.

We introduce rectangular coordinate systems (X_S, Y_S, Z_S), (x_i, y_i, z_i), and (X, Y, Z) attached to S, G_i, and $\sum$, respectively. The origins are at O_S, O_i, and O. The X_S, x_i, and X axes are the normals to the respective elements, and Y_S, y_i, and Y axes lie in the meridional plane. A ray originating from a source point $P_0(0, s, z)$ in S goes through a point $P_i(\xi_i, w_i, l_i)$ on the n_ith groove of G_i. The diffracted ray of wavelength λ in m_nth order from P_n on the last grating G_n intersects $\sum$ at a point $B(0, Y, Z)$, forming a spot in $\sum$.

The zeroth groove of G_i is assumed to pass through O_i. The groove number n_i of G_i can be expressed in a power series of w_i and l_i as[3, 4]

$$n_i\sigma_i = w_i + (n_i)_{20}w_i^2 + (n_i)_{02}l_i^2 + (n_i)_{30}w_i^3 + (n_i)_{12}w_il_i^2 + (n_i)_{40}w_i^4 + (n_i)_{22}w_i^2l_i^2 + (n_i)_{04}l_i^4 + \cdots, \tag{1}$$

where σ_i is the effective grating constant[4] of G_i. For explicit expressions of the groove parameters $(n_i)_{jk}$ of advanced gratings, refer to Refs. 3 and 4.

The surface figure of G_i is given by

$$\xi_i = a_i - a_i\sqrt{1-\left(\frac{w_i^2}{b_i^2}+\frac{l_i^2}{c_i^2}\right)} \tag{2}$$

where a_i, b_i, and c_i are the ellipsoidal semiaxes of G_i with respect to the x_i, y_i, and z_i axes, respectively.

The angles of incidence α_i and diffraction β_i of the principal ray at G_i are taken as positive or negative according to whether they are defined in the first quadrant or in the fourth quadrant of the x_iy_i plane. The angles α_i and β_i are related through the grating equation

$$\sigma_i(\sin\alpha_i + \sin\beta_i) = m_i\lambda \tag{3}$$

and the sign of m_i is defined by Eq. (3) and the sign convention for α_i and β_i.

The image plane Σ is expressed in the $x_ny_nz_n$ coordinate system as

$$x'\cos\beta_n + y'\sin\beta_n = r \tag{4}$$

where $r = O_nO$, and the coordinates (x', y', z') of a point in Σ are defined in the $x_ny_nz_n$ coordinate system.

3 Ray – tracing formalism for a multigrating system

Application of Fermat's principle to the light path function

$$F_1 = P_0P_1 + P_1P_2 + n_1m_1\lambda \tag{5}$$

for G_1 yields the direction cosines (L'_1, M'_1, N'_1) of the diffracted ray P_1P_2 from G_1 in terms of the direction cosines (L_1, M_1, N_1) of the incident ray P_0P_1 onto G_1 and given instrumental parameters[4] (see Fig. 1):

$$L'_1 = L_1 + t_1,\ M'_1 = M_1 + m_1\lambda\left(\frac{\partial n_1}{\partial w_1}\right) - t_1\left(\frac{\partial \xi_1}{\partial w_1}\right),$$

$$N'_1 = N_1 + m_1\lambda\left(\frac{\partial n_1}{\partial l_1}\right) - t_1\left(\frac{\partial \xi_1}{\partial l_1}\right) \tag{6}$$

where all the quantities are defined in the $x_1y_1z_1$ coordinate system. In Eq. (6), t_1 is expressed as

$$t_1 = \frac{1}{e_1}\left(p_1 + \sqrt{p_1^2 - e_1q_1}\right) \tag{7}$$

where

$$e_1 = 1 + \left(\frac{\partial \xi_1}{\partial w_1}\right)^2 + \left(\frac{\partial \xi_1}{\partial l_1}\right)^2$$

$$p_1 = -L_1 + \left(M_1 + m_1\lambda\frac{\partial n_1}{\partial w_1}\right)\frac{\partial \xi_1}{\partial w_1} + \left(N_1 + m_1\lambda\frac{\partial n_1}{\partial l_1}\right)\frac{\partial \xi_1}{\partial l_1}$$

$$q_1 = 2m_1\lambda\left(M_1\frac{\partial n_1}{\partial w_1} + N_1\frac{\partial n_1}{\partial l_1}\right) + (m_1\lambda)^2\left[\left(\frac{\partial n_1}{\partial w_1}\right)^2 + \left(\frac{\partial n_1}{\partial l_1}\right)^2\right] \tag{8}$$

Here, the direction cosines L_1, M_1, N_1 are defined by

$$L_1 = \frac{\xi_1 - x_0}{P_0P_1},\ M_1 = \frac{w_1 - y_0}{P_0P_1},\ N_1 = \frac{l_1 - z_0}{P_0P_1} \tag{9}$$

where $x_0 = r_1\cos\alpha_1 + s\sin\alpha_1$, $y_0 = r_1\sin\alpha_1 - s\cos\alpha_1$, and $z_0 = z$ are the coordinates of the point P_0 defined in the $x_1y_1z_1$ coordinate system.

The intersecting point $P_2(\xi_2, w_2, l_2)$ defined in the $x_2y_2z_2$ coordinate system is determined by simultaneously solving the equation of the ray P_1P_2

$$\frac{\xi_2 - \xi_1}{L_2} = \frac{w_2 - \overline{w_1}}{M_2} = \frac{l_2 - \overline{l}_1}{N_2} \tag{10}$$

and Eq. (2) with $i = 2$. In Eq. (10), $\xi_1\overline{w_1}\overline{l}_1$ and (L_2, M_2, N_2) are the coordinates of the point P_1 and the direction cosines of the diffracted ray P_1P_2, both defined in the $x_2y_2z_2$ coordinate system, and they are obtained by applying proper coordinate transformations to (ξ_1, w_1, l_1) and (L'_1, M'_1, N'_1). The direction cosines (L'_2, M'_2, N'_2) of the ray P_2P_3 diffracted from G_2 are obtained from Eqs. (6)–(8) by replacement of the subscript 1 with 2.

Following the same procedure we can determine the direction cosines (L_i, M_i, N_i) of the incident ray $P_{i-1}P_i$ and (L'_i, M'_i, N'_i) of the diffracted ray P_iP_{i+1} at G_i successively from $i = 3$ to $i = n$. Then, the intersection $B(x_B, y_B, z_B)$ of the diffracted ray P_nB with the image plane $\sum$ is determined by simultaneously solving Eq. (4) and the equation of the ray P_nB

$$(x_B - \xi_n)/L'_n = (y_B - w_n)/M'_n = (z_B - l_n)/N'_n \tag{11}$$

after replacing x' and y' in Eq. (4) with x_B and y_B, respectively.

The result is given by

$$x_B = \xi_n + L'_nK,\ y_B = w_n + M'_nK,\ z_B = l_n + N'_nK \tag{12}$$

$$K = \frac{r - \zeta_n\cos\beta_n - w_n\sin\beta_n}{L'_n\cos\beta_n + M'_n\sin\beta_n} \tag{13}$$

Finally, the ray–traced spot $B(0, Y, Z)$ in the XYZ coordinate system is expressed as

$$Y = (r\sin\beta_n - y_B)\sec\beta_n,\ Z = z_B \tag{14}$$

The equations given in this section provide a complete set of the ray–tracing formulas for the n–grating system.

4 Spot-diagram formulas

We derive analytic expressions of the direction cosines (L_i, M_i, N_i) of the incident ray $P_{i-1}P_i$ and (L'_i, M'_i, N'_i) of the diffracted ray P_iP_{i+1} at G_i successively from $i = 1$ to $i = 3$ by analytically following the ray-tracing formalism described in section 3 with the aid of power series expansions done by Mathematica$^{\mathrm{TM}}$. The result obtained for $i = 1$, 2, and 3 is expressed as follows in the form of power series of $w^h l^j s^k z^q$ with $h + j + k + q \leqslant 3$:

$$\begin{aligned} L_i = {}& -\cos\alpha_i + w_i(L_i)_{1000} + s(L_i)_{0010} + \\ & w_i^2(L_i)_{2000} + w_i s(L_i)_{1010} + l_i^2(L_i)_{0200} + \\ & l_i z(L_i)_{0101} + s^2(L_i)_{0020} + z^2(L_i)_{0002} + \\ & w_i^3(L_i)_{3000} + w_i^2 s(L_i)_{2010} + \\ & w_i l_i^2(L_i)_{1200} + w_i l_i z(L_i)_{1101} + \\ & w_i s^2(L_i)_{1020} + w_i z^2(L_i)_{1002} + l_i^2 s(L_i)_{0210} + \\ & l_i s z(L_i)_{0111} + s^3(L_i)_{0030} + s z^2(L_i)_{0012} \end{aligned} \qquad (15a)$$

$$\begin{aligned} M_i = {}& -\sin\alpha_i + w_i(M_i)_{1000} + s(M_i)_{0010} + \\ & w_i^2(M_i)_{2000} + w_i s(M_i)_{1010} + l_i^2(M_i)_{0200} + \\ & l_i z(M_i)_{0101} + s^2(M_i)_{0020} + z^2(M_i)_{0002} + \\ & w_i^3(M_i)_{3000} + w_i^2 s(M_i)_{2010} + \\ & w_i l_i^2(M_i)_{1200} + w_i l_i z(M_i)_{1101} + \\ & w_i s^2(M_i)_{1020} + w_i z^2(M_i)_{1002} + l_i^2 s(M_i)_{0210} + \\ & l_i s z(M_i)_{0111} + s^3(M_i)_{0030} + s z^2(M_i)_{0012} \end{aligned} \qquad (15b)$$

$$\begin{aligned} N_i = {}& l_i(N_i)_{0100} + z(N'_i)_{0001} + w_i l_i(N'_i)_{1100} + \\ & w_i z(N_i)_{1001} + l_i s(N_i)_{0110} + \\ & w_i^2 l_i(N_i)_{2100} + w_i^2 z(N_i)_{2001} + w_i l_i s(N_i)_{1110} + \\ & w_i s z(N_i)_{1011} + l_i^3(N_i)_{0300} + \\ & l_i^2 z(N_i)_{0201} + l_i s^2(N_i)_{0120} + l_i z^2(N_i)_{0102} + \\ & s^2 z(N_i)_{0021} + z^3(N_i)_{0003} \end{aligned} \qquad (15c)$$

$$\begin{aligned} L'_i = {}& -\cos\beta_i + w_i(L'_i)_{1000} + s(L'_i)_{0010} + \\ & w_i^2(L'_i)_{2000} + w_i s(L'_i)_{1010} + l_i^2(L'_i)_{0200} + \\ & l_i z(L'_i)_{0101} + s^2(L'_i)_{0020} + z^2(L'_i)_{0002} + \\ & w_i^3(L'_i)_{3000} + w_i^2 s(L'_i)_{2010} + \\ & w_i l_i^2(L'_i)_{1200} + w_i l_i z(L'_i)_{1101} + \\ & w_i s^2(L'_i)_{1020} + w_i z^2(L'_i)_{1002} + l_i^2 s(L'_i)_{0210} + \\ & l_i s z(L'_i)_{0111} + s^3(L'_i)_{0030} + s z^2(L'_i)_{0012} \end{aligned} \qquad (16a)$$

$$\begin{aligned} M'_i = {}& -\sin\beta_i + w_i(M'_i)_{1000} + s(M'_i)_{0010} + \\ & w_i^2(M'_i)_{2000} + w_i s(M'_i)_{1010} + l_i^2(M'_i)_{0200} + \\ & l_i z(M'_i)_{0101} + s^2(M'_i)_{0020} + z^2(M'_i)_{0002} + \\ & w_i^3(M'_i)_{3000} + w_i^2 s(M'_i)_{2010} + \\ & w_i l_i^2(M'_i)_{1200} + w_i l_i z(M'_i)_{1101} + \\ & w_i s^2(M'_i)_{1020} + w_i z^2(M'_i)_{1002} + l_i^2 s(M'_i)_{0210} + \\ & l_i s z(M'_i)_{0111} + s^3(M'_i)_{0030} + s z^2(M'_i)_{0012} \end{aligned} \qquad (16b)$$

$$\begin{aligned} N'_i = {}& L_i(N'_i)_{0100} + z(N'_i)_{0001} + w_i l_i(N'_i)_{1100} + \\ & w_i z(N'_i)_{1001} + l_i s(N'_i)_{0110} + \\ & w_i^2 l_i(N'_i)_{2100} + w_i^2 z(N'_i)_{2001} + w_i l_i s(N'_i)_{1110} + \\ & w_i s z(N'_i)_{1011} + l_i^3(N'_i)_{0300} + \\ & l_i^2 z(N'_i)_{0201} + l_i s^2(N'_i)_{0120} + l_i z^2(N'_i)_{0102} + \\ & s^2 z(N'_i)_{0021} + z^3(N'_i)_{0003} \end{aligned} \qquad (16c)$$

For $i = 1$, special attention should be paid to the fact that the terms $l_1 s\ (N_1)_{0110}$ and $s z\ (N_1)_{0011}$ are absent in Eq. (15c).

In deriving Eqs. (15) and (16) we used the following relations between (w_{i-1}, l_{i-1}) and (w_i, l_i), which were derived for $i = 2$ and 3 by following the procedure given in Refs. 3 and 4:

$$\begin{aligned} w_{i-1} = {}& w_i(U_i)_{1000} + s(U_i)_{0010} + w_i^2(U_i)_{2000} + \\ & w_i s(U_i)_{1010} + l_i^2(U_i)_{0200} l_i z(U_i)_{0101} + s^2(U_i)_{0020} + \\ & z^2(U_i)_{0002} + w_i^3(U_i)_{3000} + w_i^2 s(U_i)_{2010} + \\ & w_i l_i^2(U_i)_{1200} + w_i l_i z(U_i)_{1101} + w_i s^2(U_{i1020} + \\ & w_i z^2(U_i)_{1002} + l_i^2 s(U_i)_{0210} + \\ & l_i s z(U_i)_{0111} + s^3(U_i)_{0030} + s z^2(U_i)_{0012} \end{aligned} \qquad (17a)$$

$$\begin{aligned} l_{i-1} = {}& l_i(V_i)_{0100} + z(V_i)_{0001} + w_i l_i(V_i)_{1100} + \\ & w_i z(V_i)_{1001} + l_i s(V_i)_{0110} + s z(V_i)_{0011} + \\ & w_i^2 l_i(V_i)_{2100} + w_i^2 z(V_i)_{2001} + w_i l_i s(V_i)_{1110} + \\ & w_i s z(V_i)_{1011} + l_i^3(V_i)_{0300} + \\ & l_i^2 z(V_i)_{0201} + l_i s^2(V_i)_{0120} + l_i z^2(V_i)_{0102} + \end{aligned}$$

$$s^2 z(V_i)_{0021} + z^3(V_i)_{0003} \qquad (17b)$$

$$\begin{aligned}w_1 = {} & w_{i-1}(E_{i-1})_{1000} + s(E_{i-1})_{0010} + w_{i-1}^2 (E_{i-1})_{2000} + w_{i-1}s(E_{i-1})_{1010} + l_{i-1}^2(E_{i-1})_{0200} + \\ & l_{i-1}z(E_{i-1})_{0101} + s^2(E_{i-1})_{0020} + z^2(E_{i-1})_{0002} + w_{i-1}^3(E_{i-1})_{3000} + w_{i-1}^2 s(E_{i-1})_{2010} + \\ & w_{i-1}l_{i-1}^2(E_{i-1})_{1200} + w_{i-1}l_{i-1}z(E_{i-1})_{1101} + w_{i-1}s^2(E_{i-1})_{1020} + w_{i-1}z^2(E_{i-1})_{1002} + \\ & l_{i-1}^2 s(E_{i-1})_{0210} + l_{i-1}sz(E_{i-1})_{0111} + s^3(E_{i-1})_{0030} + sz^2(E_{i-1})_{0012} \qquad (18a)\end{aligned}$$

$$\begin{aligned}l_i = {} & l_{i-1}(F_{i-1})_{0100} + z(F_{i-1})_{0001} + w_{i-1}l_{i-1}(F_{i-1})_{1100} + w_{i-1}z(F_{i-1})_{1001} + l_{i-1}s(F_{i-1})_{0110} + \\ & sz(F_{i-1})_{0011} + w_{i-1}^2 l_{i-1}(F_{i-1})_{2100} + w_{i-1}^2 z(F_{i-1})_{2001} + w_{i-1}l_{i-1}s(F_{i-1})_{1110} + \\ & w_{i-1}sz(F_{i-1})_{1011} + l_{i-1}^3(F_{i-1})_{0300} + l_{i-1}^2 z(F_{i-1})_{0201} + l_{i-1}s^2(F_{i-1})_{0120} + \\ & l_{i-1}z^2(F_{i-1})_{0102} + s^2 z(F_{i-1})_{0021} + z^3(F_{i-1})_{0003} \qquad (18b)\end{aligned}$$

For explicit expressions of $(U_i)_{hjkq}$, $(V_i)_{hjkq}$, $(E_{i-1})_{hjkq}$, and $(F_{i-1})_{hjkq}$ for a double-grating system refer to Refs. 1 and 2.

We now assume that Eqs. (15) and (16) hold for the ith element, i being greater than 3. Then, we follow the ray-tracing formalism again to derive the direction cosines of the rays P_iP_{i+1} and $P_{i+1}P_{i+2}$ in terms of w_{i+1} and l_{i+1} after determining the relations between (w_i, l_i) and (w_{i+1}, l_{i+1}). It is easily shown that Eqs. (15)-(18) also hold for the $(i+1)$th element. Therefore, this procedure based on the mathematical induction proves that Eqs. (15)-(18) hold for any element with $n \geqslant i \geqslant 2$.

The diffracted ray of wavelength λ in m_nth order from P_n intersects the image plane Σ at a point $B(0, Y, Z)$. Using the expressions of the direction cosines (L'_n, M'_n, N'_n) of the ray P_nB, Eqs. (16) with $i = n$, we can calculate the coordinates of the spot B from Eqs. (12)-(14). The resulted expressions for Y and Z are given by

$$\begin{aligned}Y = {} & w_n(C_n)_{1000} + s(C_n)_{0010} + w_n^2(C_n)_{2000} + w_n s(C)_{1010} + l_n^2(C)_{0200} + \\ & l_n z(C)_{0101} + s^2(C_n)_{0020} + z^2(C_n)_{0002} + w_n^3(C_n)_{3000} + w_n^2 s(C_n)_{2010} + \\ & w_n l_n^2(C_n)_{1200} + w_n l_n z(C)_{1101} + w_n s^2(C_n)_{1020} + w_n z^2(C_n)_{1002} + l_n^2 s(C_n)_{0210} + \\ & l_n sz(C_n)_{0111} + s^3(C_n)_{0030} + sz^2(C_n)_{0012} \qquad (19a)\end{aligned}$$

$$\begin{aligned}Z = {} & l_n(D_n)_{0100} + z(D_n)_{0001} + w_n l_n(D_n)_{1100} + w_n z(D_n)_{1001} + l_n s(D_n)_{0110} + sz(D_n)_{0011} + \\ & w_n^2 l_n(D_n)_{2100} + w_n^2 z(D_n)_{2001} + w_n l_n s(D_n)_{1110} + w_n sz(D_n)_{1011} + l_n^3(D_n)_{0300} + \\ & l_n^2 z(D_n)_{0201} + l_n s^2(D_n)_{0120} + l_n z_2(D_n)_{0102} + s^2 z(D_n)_{0021} + z^3(D_n)_{0003} \qquad (19b)\end{aligned}$$

Equations (19a, b) are the spot-diagram formulas, and $(C_n)_{hjkq}$ and $(D_n)_{hjkq}$ are the aberration coefficients of the system. The subscript $hjkq$ indicates that $(C_n)_{hjkq}$ and $(D_n)_{hjkq}$ are the coefficients of the $w^h l^j s^k z^q$ term.

Repeated substitutions of Eqs. (18a, b) into Eqs. (19a, b) by taking i from n to 2 yield the result that the spot diagram formulas of the system can be expressed as functions of w_i and l_i of the ith element that have the same form as Eqs. (19a, b) with coefficients $(C_i)_{hjkq}$ and $(D_i)_{hjkq}$ in place of $(C_n)_{hjkq}$ and $(D_n)_{hjkq}$. This is the important conclusion in treating multielement systems. In other words, if we obtain the spot-diagram formulas for double-element system, we can express the spot-diagram formulas for a multielement system in the same forms as those for the double-element system. For example, suppose we found a relation $(C_i)_{2000} = (C_i)_{1010} = (C_i)_{0200} = (C_i)_{0101} = (C_i)_{0020} = (C_i)_{0002} = (D_i)_{1100} = (D_i)_{1001} = (D_i)_{0110} = (D_i)_{0011} = 0$ with $i = 1, 2$ for an axially symmetric double-mirror system ($m_1 = m_2 = \alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$, $R_1 = \rho_1$, and $R_2 = \rho_2$)[1]. Then, we can immediately write down spot diagram formulas for an axially symmetric multimirror system as

$$Y = w_i(C_i)_{1000} + s(C_i)_{0010} + w_i^3(C_i)_{3000} +$$

$$w_i^2s(C_i)_{2010}+w_il_i^2(C_i)_{1200}+ w_il_iz(C_i)_{1101}+w_is^2(C_i)_{1020}+ w_iz^2(C_i)_{1002}+l_i^2s(C_i)_{0210}+ l_isz(C_i)_{0111}+s^3(C_i)_{0030}+sz^2(C_i)_{0012} \tag{20a}$$

$$Z= l_i(D_i)_{0100}+z(D_i)_{0001}+w_i^2l_i(D_i)_{2100}+ w_i^2z(D_i)_{2001}+w_il_is(D_i)_{1110}+ w_isz(D_i)_{1011}+l_i^3(D_i)_{0300}+ l_i^2z(D_i)_{0201}+l_is^2(D_i)_{0120}+ l_iz^2(D_i)_{0102}+s^2z(D_i)_{0021}+z^3(D_i)_{0003} \tag{20b}$$

Explicit expressions of the coefficients in Eqs. (15) ((19) have been obtained for the single grating[4] ($n = 1$) and double- grating system[1- 2] ($n = 2$). However, an extension to $n > 2$ requires an incalculable amount of time and effort even using Mathematica™. A possible solution to this problem is to combine the versatility of ray tracing with the capability of analytic expressions in giving physical insight. We employed this approach[5] to make the geometric theory practical for the design and aberration analysis of a multielement optical system.

For a chosen set of s and z we determine the values of w_i, l_i, Y, and Z by tracing N ($\geqslant 18$) rays through the system and make N equations upon substitution of these values into Eqs. (19a, b) with $n = i$. The probable values of $(C_i)_{hjkq}$ and $(D_i)_{hjkq}$ are determined as the regression coefficients of Eqs. (19a, b) when $N > 18$ or as the solution of the simultaneous equations when $N = 18$. Then, the spot diagrams (Y, Z) are represented analytically as functions of w_i and l_i with numerical coefficients. The equations of individual aberration curves are then formulated in a similar manner to Seidel aberrations of lens systems.

5 Third- order aberration curves

We give in this section the equations of aberration curves for coma, spherical aberration, and resultant aberration of a multigrating system.

A. Coma

Coma of a multigrating system is represented by

$$Y_{\text{coma}} = (c_i)_{2000}w_i^2+(C_i)_{2010}w_i^2s+ (C_i)_{0210}l_i^2s+(C_i)_{1101}w_il_iz \tag{21a}$$

$$Z_{\text{coma}} = (D_i)_{1100}w_il_i+(D_i)_{1110i}L_is+ (D_i)_{2001}w_i^2z+(D_i)_{0201}l_i^2z \tag{21b}$$

Here, the subscript "coma" signifies the contribution from coma only, assuming all the other aberrations are absent. Introduction of polar coordinates

$$w_i = \tilde{r}\cos\theta,\quad l_i = \tilde{r}\sin\theta \tag{22}$$

in the tangent plane of G_i at its vertex O_i transforms Eqs. (21a, b) into

$$\frac{2Y_{\text{coma}}}{\tilde{r}^2}-\{(C_i)_{2000}+s[(C_i)_{2010}+(C_i)_{0210}]\} = \{(C_i)_{2000}+s[(C_i)_{2010}-(C_i)_{0210}]\} \cos2\theta+z(C_i)_{1101}\sin2\theta \tag{23a}$$

$$\frac{2Z_{\text{coma}}}{\tilde{r}^2}-z[(D_i)_{2001}+(D_i)_{0201}] = z[(D_i)_{2001}-(D_i)_{0201}]\cos2\theta+ [(D_i)_{1100}+s(D_i)_{1110}]\sin2\theta \tag{23b}$$

where θ is measured counterclockwise from the positive y_i axis.

Eliminating $\sin2\theta$ and $\cos2\theta$ from Eqs. (23a, b), we obtain

$$a\left\{\frac{2Y_{\text{coma}}}{\tilde{r}^2}-\{(C_i)_{2000}+s[(C_i)_{2010}+(C_i)_{0210}]\}\right\}^2+ b\left[\frac{2Z_{\text{coma}}}{\tilde{r}^2}-z[(D_i)_{2001}+(D_i)_{0201}]\right]^2- 2h\left\{\frac{2Y_{\text{coma}}}{\tilde{r}^2}-\{(C_i)_{2000}+s[(C_i)_{2010}+(C_i)_{0210}]\}\right\}\times \left\{\frac{2Z_{\text{coma}}}{\tilde{r}^2}-z[(D_i)_{2001}+(D_i)_{0201}]\right] = c^2 \tag{24}$$

where

$$a = [(D_i)_{1100}+s(D_i)_{1110}]^2+ z^2[(D_i)_{2001}-(D_i)_{0201}]^2$$

$$b = \{(C_i)_{2000}+s[(C_i)_{2010}-(C_i)_{0210}]\}^2+ z^2[(C_i)_{1101}]^2$$

$$c = \{(C_i)_{2000} + s[(C_i)_{2010} - (C_i)_{0210}]\}$$
$$[(D_i)_{1100} + s(D_i)_{1110}] - z^2(C_i)_{1101}$$
$$[(D_i)_{2001} - (D_i)_{0201}]$$
$$h = z\{(C_i)_{1101}[(D_i)_{1100} + s(D_i)_{1110}] +$$
$$[(C_i)_{2000} + s((C_i)_{2010} - (C_i)_{0210})]$$
$$[(D_i)_{2001} - (D_i)_{0201}]\} \qquad (25)$$

Equation (25) represents an ellipse because $h^2 - ab < 0$. Therefore, for fixed values of s, z, and $\tilde{r}$, the point B describes an ellipse twice as θ runs from 0 through 2π.

We consider an axially symmetric multimirror system as a special case of the multigrating system. We further consider a case where radiating source points in S are localized around a point $A(0, 0, z_A)$ on the Z_S axis and the ray originating from A impinges on G_1 at a point $Q_1(\xi_1, 0, \zeta_1)$ in the $x_1y_1z_1$ coordinate system. The chief ray AQ_1 goes through G_i *at* $Q_i(\xi_i, 0, \zeta_i)$ in the $x_iy_iz_i$ coordinate system. In this case, Eqs. (22) should be replaced by

$$w_i = \tilde{r}\cos\theta, \quad l_i = \zeta_i + \tilde{r}\sin\theta \qquad (26)$$

where θ is measured counterclockwise from the initial line parallel to the y_i axis and the pole of the polar coordinates is at the projection of Q_i onto the tangent plane of G_i at O_i. In this case, Eqs. (23) – (25) must be modified accordingly. Note here that $(C_i)_{2000} = (D_i)_{1100} = 0$ and coma curves are not ellipses. Projection optics for EUV lithography is an example of such a case.

B. Spherical aberration

Spherical aberration is described by

$$Y_{\mathrm{sph}} = (C_i)_{3000} w_i^3 + (C_i)_{1200} w_i l_i^2 \qquad (27a)$$

$$Z_{\mathrm{sph}} = (D_i)_{0300} l_i^3 + (D_i)_{2100} w_i^2 l_i \qquad (27b)$$

Here, the subscript "sph" signifies the contribution from spherical aberration only, assuming all the other aberratios are absent.

Substitution of Eq. (22) in Eqs. (27a, b) yields

$$Y_{\mathrm{sph}} = r^3\cos\theta[(C_i)_{3000}\cos^2\theta +$$
$$(C_i)_{1200}\sin^2\theta] \qquad (28a)$$

$$Z_{\mathrm{sph}} = r^3\sin\theta[(D_i)_{0300}\sin^2\theta +$$
$$(D_i)_{2100}\cos^2\theta] \qquad (28b)$$

This aberration curve takes a complicated form that depends on the values of $(C_i)_{3000}$, $(C_i)_{1200}$, $(D_i)_{0300}$, and $(D_i)_{2100}$. In case of necessity, use Eqs. (26) with Eqs. (27a, b).

C. Resultant aberration

The resultant aberration of the system is given by the sum of the individual aberrations. It is practical to express this aberration in terms of the last element G_n, i. e., by Eqs. (19a, b). Substitution of Eqs. (22) in Eqs. (19a, b) yields the equation of the resultant aberration curve. When necessary, use Eqs. (26) with Eqs. (19a, b).

In many cases, individual aberrations can be balanced one another to yield a small resultant aberration. The purpose of optical designs is to achieve this aberration balancing as much as possible, besides making individual aberrations small. It is therefore important to define a merit function appropriate to a given design goal.

6 Merit function

Since Eqs. (19a, b) include all the aberrations of the system up to third order, these equations would provide an effective means of optimizing aberrations in the design of a multielement system. To utilize Eqs. (19a, b) in the design work, we define a merit function so as to incorporate (1) the variance of the spots formed when an infinite number of rays are traced and (2) the dimensions of the source and the last optical element in the system. The merit function Q thus defined is expressed as

$$Q = \sum_p \varepsilon_p Q(\lambda_p) \qquad (29)$$

where λ_p is a design wavelength chosen in the required scanning range and its vicinity, ε_p is a weighting factor, and

$$Q(\lambda_p) = \frac{1}{wlbh}\int_{-W/2}^{W/2} \mathrm{d}w \int_{-L/2}^{L/2} \mathrm{d}l \int_{-B/2}^{B/2} \mathrm{d}s \int_{-H/2}^{H/2}$$
$$\{Y(\lambda_p) - \overline{Y}(\lambda_p)]^2 + \mu_p Z(\lambda_p)^2\}\mathrm{d}z \qquad (30)$$

$$\overline{Y(\lambda_p)} = \frac{1}{wlbh}\int_{-W/2}^{W/2}\mathrm{d}w\int_{-L/2}^{L/2}\mathrm{d}l\int_{-B/2}^{B/2}\mathrm{d}s\int_{-H/2}^{H/2}Y(\lambda_p)\,\mathrm{d}z \tag{31}$$

In Eqs. (30) and (31), W and L are the width and length of a ruled area of G_n illuminated by the through rays, H and B are the height and width of the source S, and μ_p is a weighting factor.

Carrying out the integration in Eqs. (30) and (31) after substitution of Eqs. (19a, b) in $Y(\lambda_p)$ and $Z(\lambda_p)$ in Eqs. (30) and (31), we obtain

$$Q(\lambda_p) = q_Y^2(\lambda_p) + \mu_p q_z^2(\lambda_p) \tag{32}$$

where

$$\begin{aligned}
q_Y^2 = {} & W^2\left[\frac{1}{12}C_{1000}^2 + \frac{B^2}{72}\left(\frac{1}{2}C_{1010}^2 + C_{1000}C_{1020} + C_{0010}C_{2010}\right) + \frac{H^2}{72}C_{1000}C_{1002} + \right.\\
& \left.\frac{B^4}{960}(C_{1020}^2 + 2C_{2010}C_{0030}) + \frac{B^2H^2}{864}(C_{2010}C_{0012} + C_{1002}C_{1020}) + \frac{H^4}{960}C_{1002}^2\right] + \\
& L^2\left[\frac{B^2}{72}C_{0010}C_{0210} + \frac{H^2}{144}C_{0101}^2 + \frac{B^4}{480}C_{0210}C_{0030} + \frac{B^2H^2}{1728}(C_{0111}^2 + 2C_{0210}C_{0012})\right] + \\
& W^4\left[\frac{1}{180}C_{2000}^2 + \frac{1}{40}C_{1000}C_{3000} + \frac{B^2}{960}(C_{2010}^2 + 2C_{3000}C_{1020}) + \frac{H^2}{480}C_{3000}C_{1002}\right] + \\
& W^2L^2\left[\frac{1}{72}C_{1000}C_{1200} + \frac{B^2}{864}(C_{2010}C_{0210} + C_{1200}C_{1020}) + \frac{H^2}{1728}(C_{1101}^2 + 2C_{1200}C_{1002})\right] + \\
& L^4\left(\frac{C_{0200}^2}{180} + \frac{B^2}{960}C_{0210}^2\right) + \frac{W^6}{448}C_{3000}^2 + \frac{W^4L^2}{480}C_{3000}C_{1200} + \frac{W^2L^4}{960}C_{1200}^2 + \\
& B^2\left[\frac{C_{0010}^2}{12} + B^2\left(\frac{C_{0020}^2}{180} + \frac{C_{0010}C_{0030}}{40}\right) + \frac{B^4}{448}C_{0030}^2\right] + \\
& B^2H^2\left(\frac{1}{72}C_{0010}C_{0012} + \frac{B^2}{480}C_{0012}C_{0030} + \frac{H^2}{960}C_{0012}^2\right) + \frac{H^4}{180}C_{0002}^2
\end{aligned} \tag{33a}$$

$$\begin{aligned}
q_z^2 = {} & W^2\left[\frac{H^2}{144}(D_{1001}^2 + 2D_{0001}D_{2001}) + \frac{H^4}{480}D_{2001}D_{0003} + \frac{B^2H^2}{1728}(D_{1101}^2 + 2D_{2001}D_{0021})\right] + \\
& L^2\left[\frac{1}{12}D_{0100}^2 + \frac{B^2}{144}(D_{0110}^2 + 2D_{0100}D_{0120}) + \frac{H^2}{72}(D_{0010}D_{0102} + D_{0001}D_{0201}) + \right.\\
& \left.\frac{B^4}{960}D_{0120}^2 + \frac{B^2H^2}{864}(D_{0201}D_{0021} + D_{0102}D_{0120}) + \frac{H^4}{960}(D_{0102}^2 + 2D_{0201}D_{0003})\right] + \\
& L^4\left[\frac{1}{40}D_{0100}D_{0300} + \frac{B^2}{480}D_{0300}D_{0120} + \frac{H^2}{960}(D_{0201}^2 + 2D_{0300}D_{0102})\right] + \\
& W^2L^2\left[\frac{1}{144}(D_{1100}^2 + 2D_{0100}D_{2100}) + \frac{B^2}{1728}(D_{1110}^2 + 2D_{2100}D_{0120}) + \frac{H^2}{864}(D_{2001}D_{0201} + D_{2100}D_{0102})\right] + \\
& \frac{W^4}{960}(L^2D_{2100}^2 + H^2D_{2001}^2) + \frac{W^2L^4}{480}D_{2100}D_{0300} + \frac{L^6}{448}D_{0300}^2 + \\
& H^2\left[\frac{D_{0001}^2}{12} + \frac{B^2}{144}(D_{0011}^2 + 2D_{0001}D_{0021}) + \frac{H^2}{40}D_{0001}D_{0003} + \frac{B^4}{960}D_{0021}^2 + \frac{B^2H^2}{480}D_{0003}D_{0021} + \frac{H^4}{448}D_{0003}^2\right]
\end{aligned} \tag{33b}$$

In Eqs. (33a, b) we have abbreviated, for simplicity, $q_Y(\lambda_p)$, $q_Z(\lambda_p)$, $(C_n)_{hjkq}(\lambda_p)$, and $(D_n)_{hjkq}(\lambda_p)$ to q_Y, q_z, C_{hjkq}, and D_{hjkq}, respectively. If the last element is circular in shape, integration should be carried out with respect to ρ and θ after replacing w_n and l_n by $\rho\cos\theta$ and $\rho\sin\theta$, respectively.

7 Conclusion

We have developed the geometric theory of a plane-symmetric multielement optical system that consists of a planar light source, an arbitrary number of ellipsoidal gratings, and an image plane. The theory is developed on the basis of the ray- tracing formalism described in section 3 and gives analytic formulas for spot diagrams, aberration curves, and a merit function. To make the theory practicable, the aberration coefficients of the spot - diagram formulas are determined numerically with the aid of ray tracing that takes into account the angular distribution of rays originating from a given light source. The spot- diagram formulas and the equation of aberration curves are expressed in terms of the coordinates of points on a chosen element in the system. The merit function defined in the present study closely represents the variance of the spots formed when an infinite number of rays are traced and when we take into account the dimensions of the light source and the last optical element. These characteristics facilitate effective minimization of aberrations and would be useful in the design of many practical optical systems. The theory can also be applied to plane- symmetric mirror- grating or mirror systems and to axially symmetric mirror systems.

Although the present theory is limited to a plane- symmetric ellipsoidal grating system, it can be extended to include a system that does not have plane symmetry as well as any restriction on the surface figure of its elements. It is also not too difficult to include aberrations up to fifth order, though laborious. It is planned to develop such a general theory of a multielement optical system.

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